

Effect of Gyroscope and Accelerometer Datasheet Parameters on Inertial Navigation Systems Accuracy

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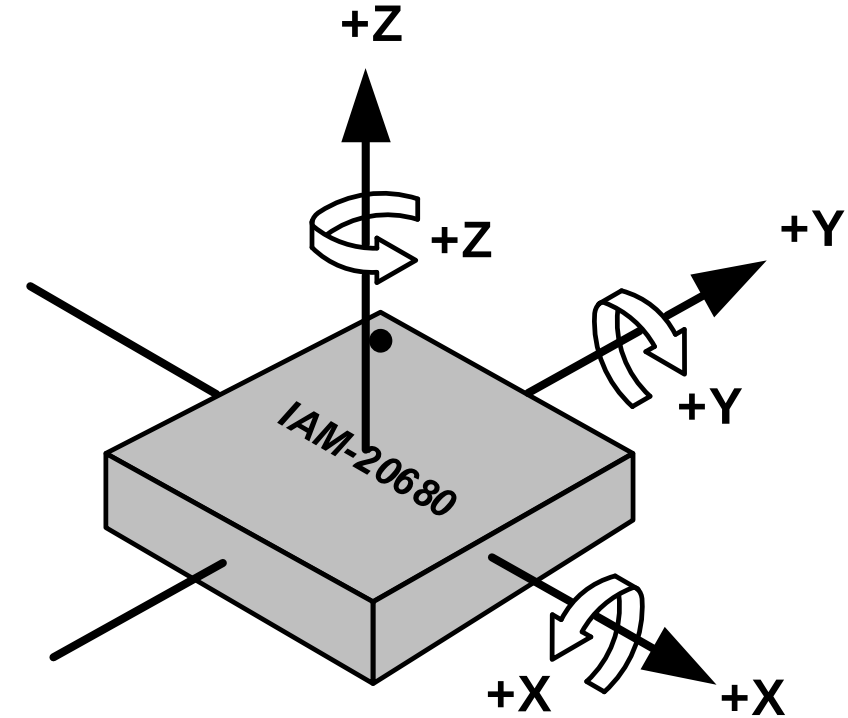
Ideal Gyroscope Model

$$Output(LSB) = S \omega$$

S = sensitivity in LSB/°/s
 ω = angular rate of rotation

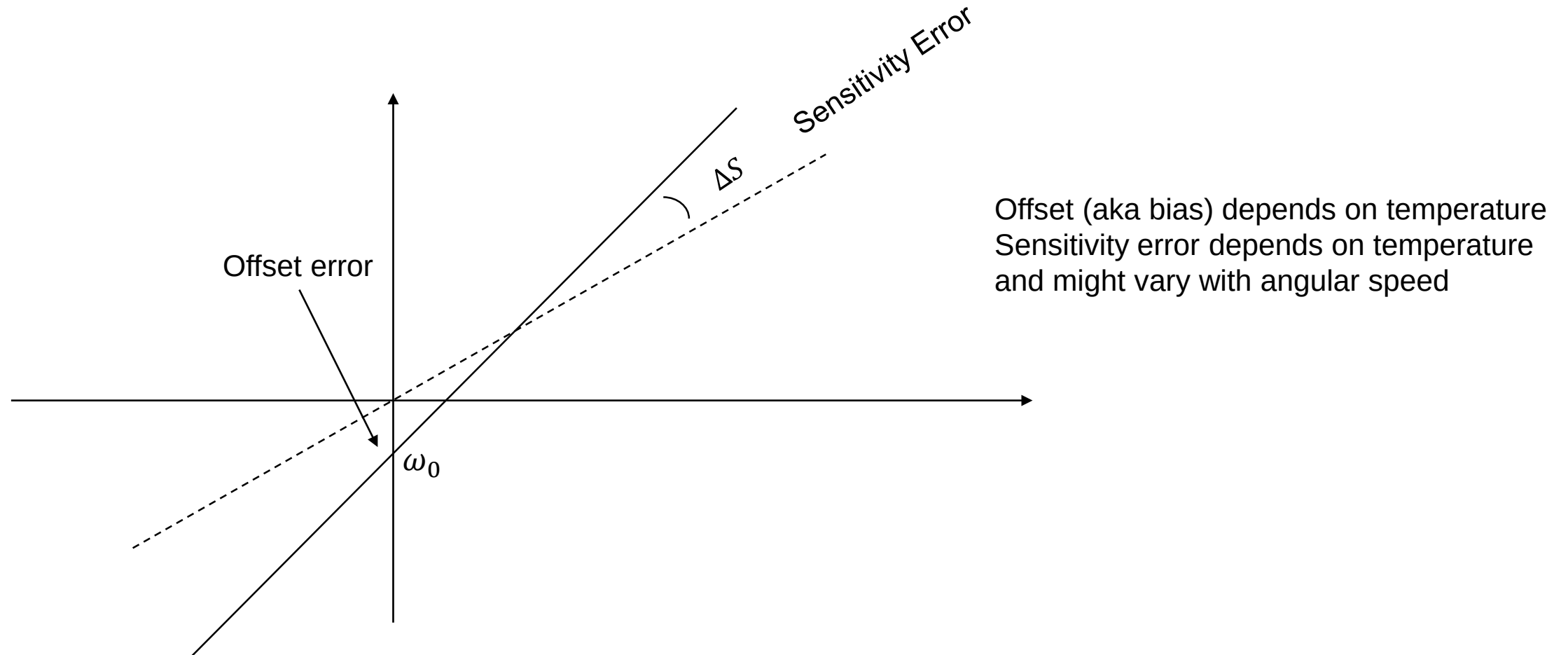
$$S = \frac{FS}{2^n}$$

FS = Full scale (typically ±125 °/s, ± 250°/s, ±500°/s, ±1000°/s, ±2000°/s)
 n = number of output bits (16 for TDK InvenSense automotive products)



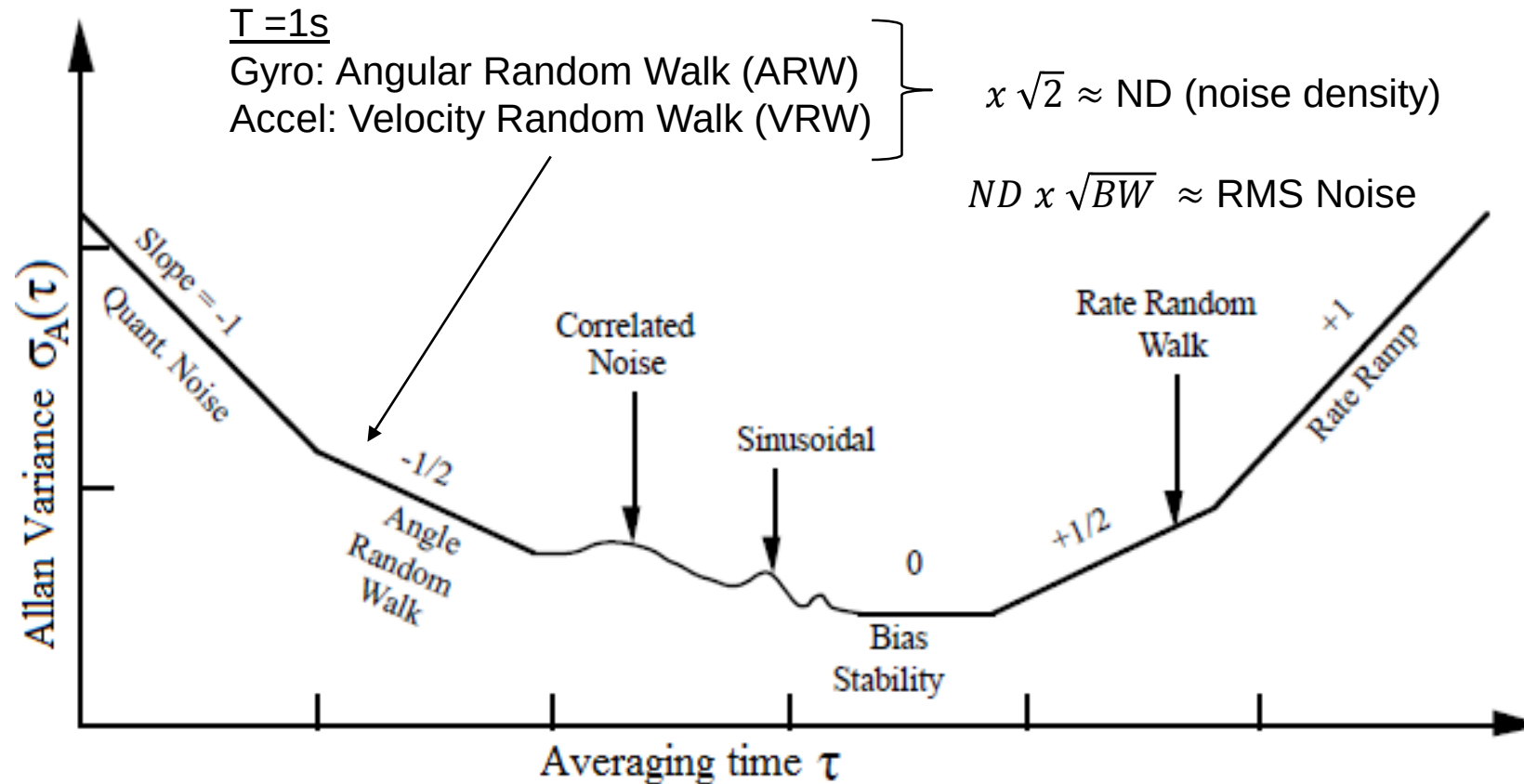
FS	S
±125°/s	262 LSB/°/s
±250°/s	131 LSB/°/s
±500°/s	65.5 LSB/°/s
±1000°/s	32.8 LSB/°/s
±2000°/s	16.4 LSB/°/s

Gyroscope Errors



$$Output(LSB) = \omega_0(T) + [S + \Delta S(\omega, T)] \omega$$

Noise – Allan Variance Plot

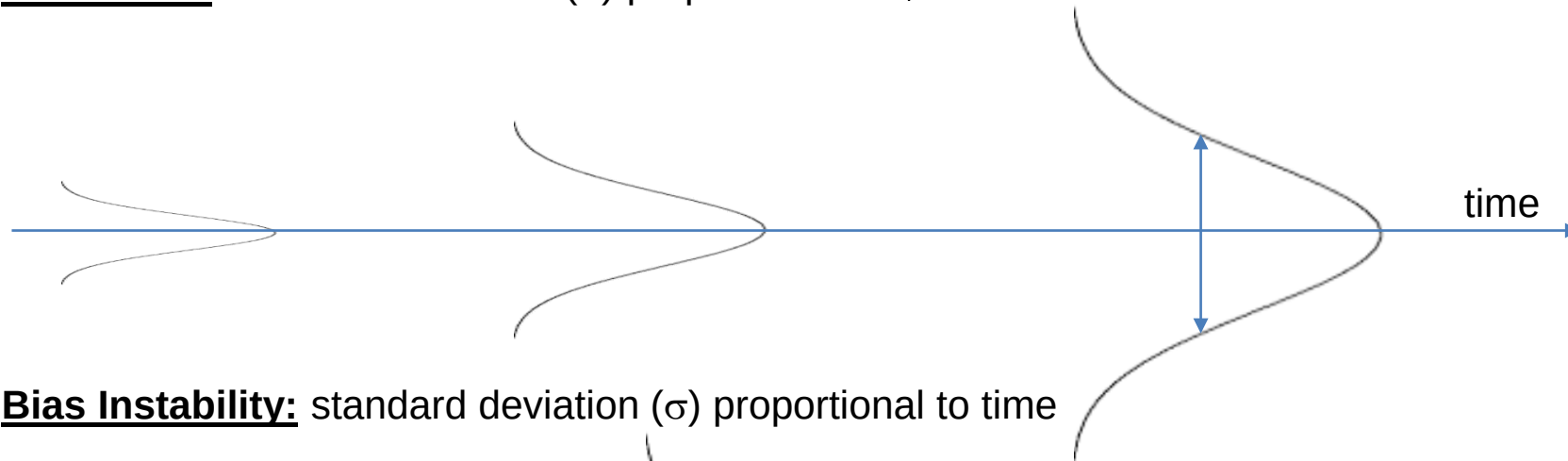


The bottom of the Allan variance curve is also known as bias instability and is related (but not necessarily identical) to the definition used by the US Department of Defense for export control purpose

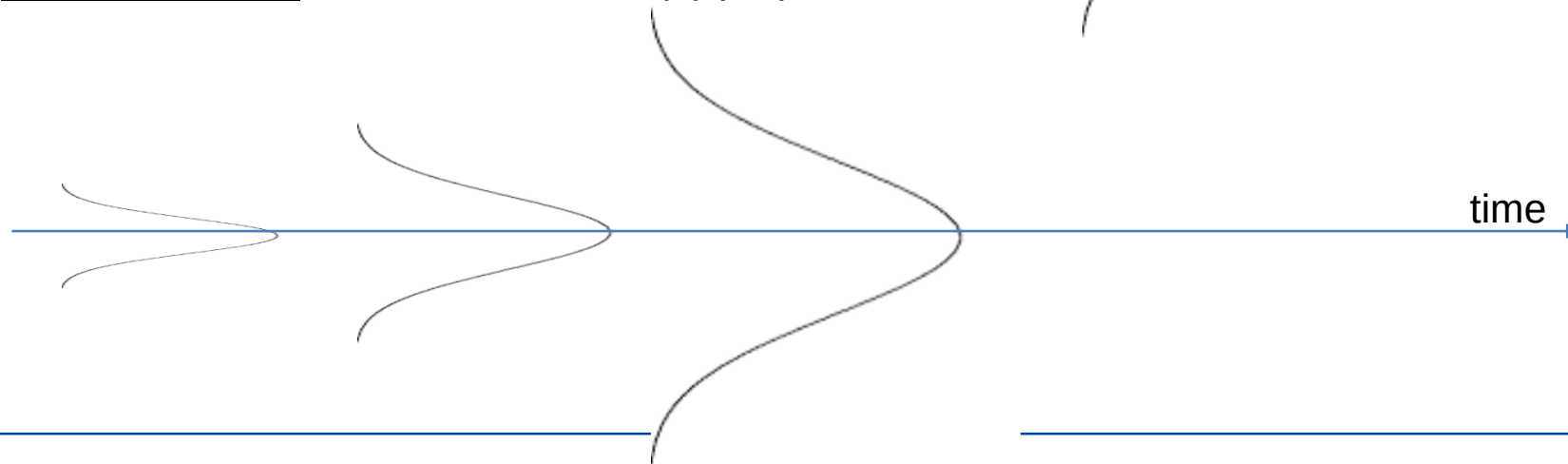
Noise Integral Over Time in IMUs

ARW/VRW and Bias Instability have zero mean, but increasing standard deviation (σ)

ARW/VRW: standard deviation (σ) proportional to $\sqrt{time}$



Bias Instability: standard deviation (σ) proportional to time



Other Errors

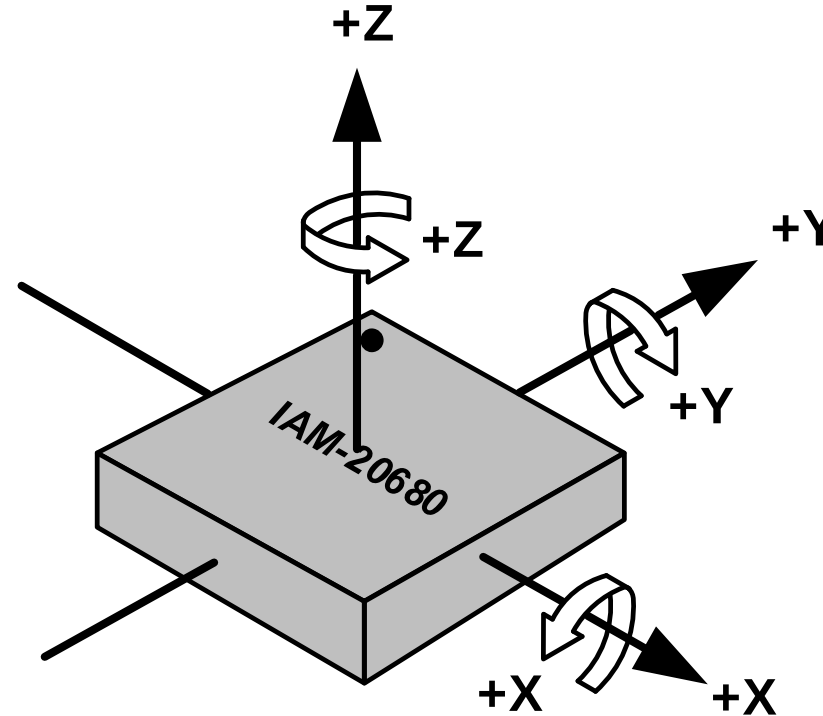
- Cross-axis
 - ↪ Impact of the rotation along one axis on the reading of the rotation on a different axis
- Non-linearity
 - ↪ $S(\omega) - \Delta S(\omega)$
 - ↪ Sometimes $S(\omega)$ can be bi-linear, e.g.
 - $S(\omega) = S_0$ for $\omega > 0$
 - $S(\omega) = S_1$ for $\omega < 0$
- Impact of acceleration
 - ↪ Acceleration and gravity affect the rotating mass in the gyroscope distorting the ideal behavior
- Vibrations
 - ↪ Vibrations can induce a small bias by distorting the behavior of the rotating mass
- Turn-on bias repeatability
 - ↪ Every time the part is turned on, the bias is just a little bit different from the last time
- Resolution
 - ↪ 12-bit accelerometer or gyroscopes may have large quantization error (with respect to noise floor)
 - ↪ TDK InvenSense current products have at least 16-bit accuracy

Accelerometer Model

$$\text{Output(}LSB\text{)} = S a$$

S = sensitivity in LSB/g
a = acceleration

$$S = \frac{FS}{2^n}$$

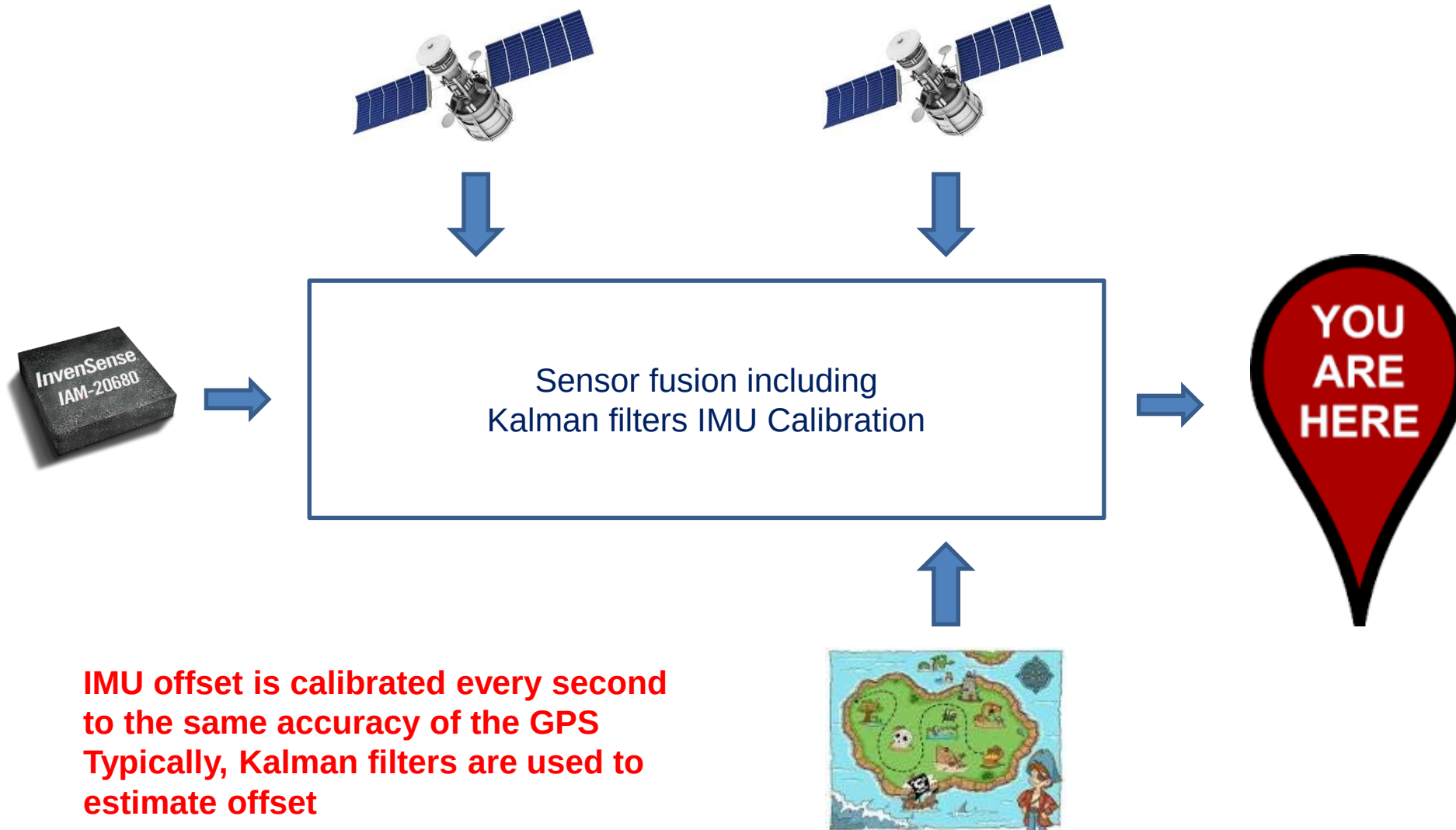


FS	S
±2g	16,384 LSB/g
±4g	8,192 LSB/g
±8g	4096 LSB/g
±16g	2048 LSB/g
±32g	1024 LSB/g

FS = Full scale (±2g, ±4g, ±8g, ±16g, ±32g, ±64g in TDK InvenSense automotive products)
N = number of output bits (16 for TDK InvenSense automotive products)

The non-idealities of an accelerometer are analogous to those of a gyroscope

Position Determination



**IMU offset is calibrated every second
to the same accuracy of the GPS
Typically, Kalman filters are used to
estimate offset**

Navigation in GNSS-denied Situations

Normally, car navigates using GNSS

Position = GNSS + map matching

What if no GNSS???

- Tunnel, parking garage, urban canyons, et .
- Inertial navigation, aka....





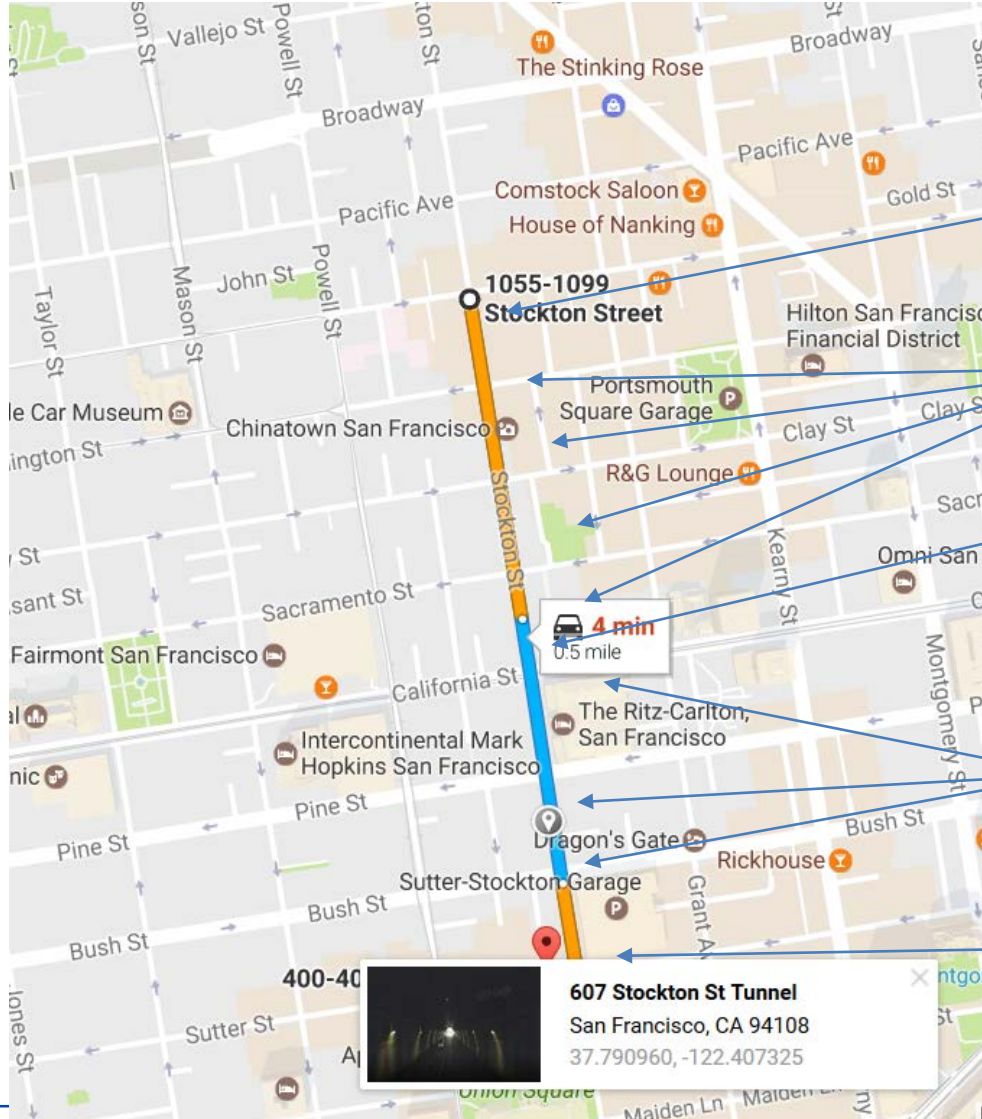
HUMPHREY
BOGART
JOHN COOMBS'S
**Dead
Reckoning**
A COLUMBIA PICTURE

Many things could be solved, if he knew why she was gambling so heavily!

What Happens Without Dead Reckoning (Video)



Navigation in GNSS-denied Environment Example



Car starts, GNSS available

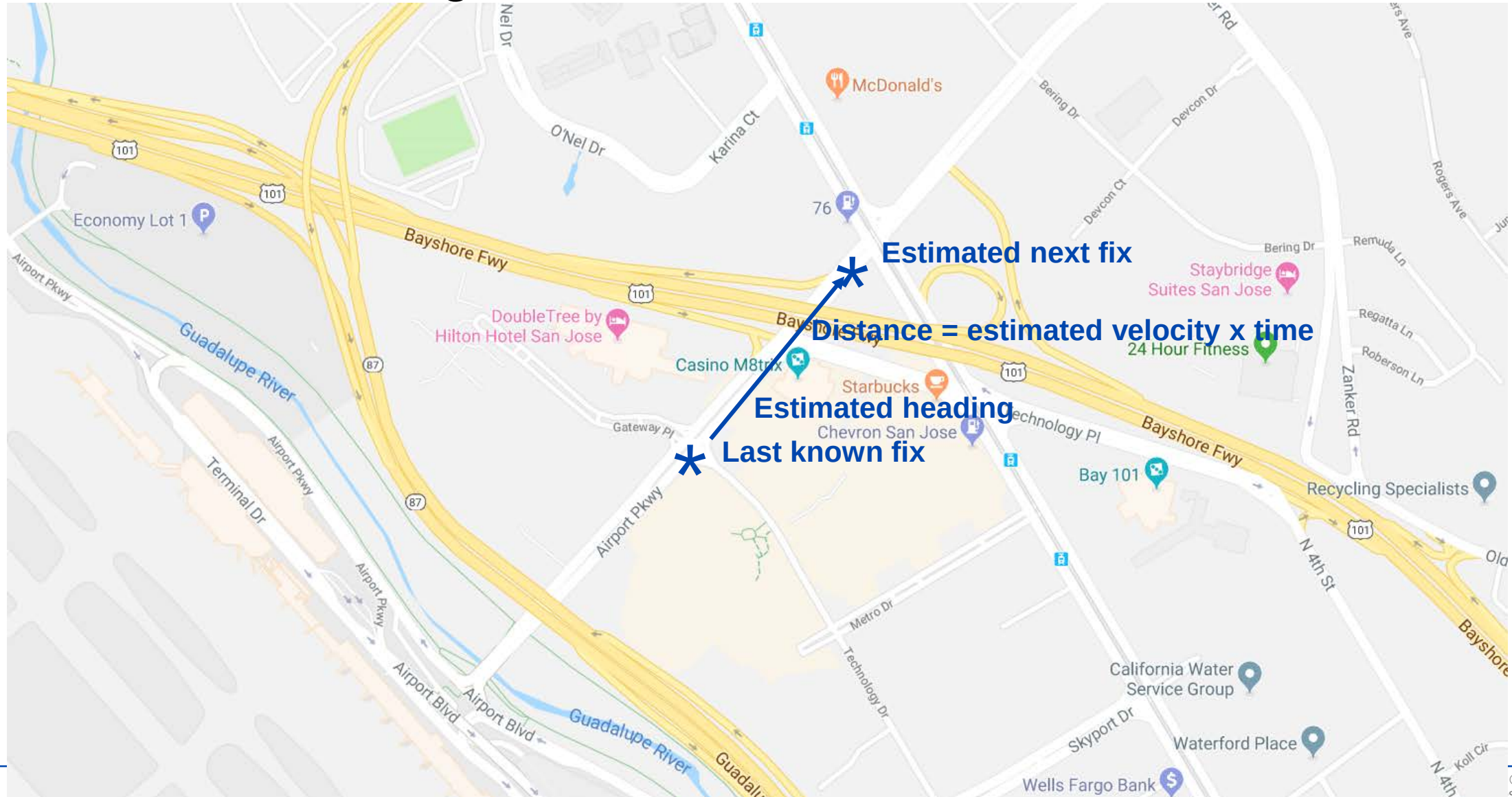
Every second the system calibrates the IMU using GNSS data

Car enters Stockton Tunnel: no GNSS, dead-reckoning begins

Noise makes bias drift, positional error increases over time, map matching and wheel ticks help dead-reckoning

Tunnel ends, GNSS resumes

Basic Dead Reckoning

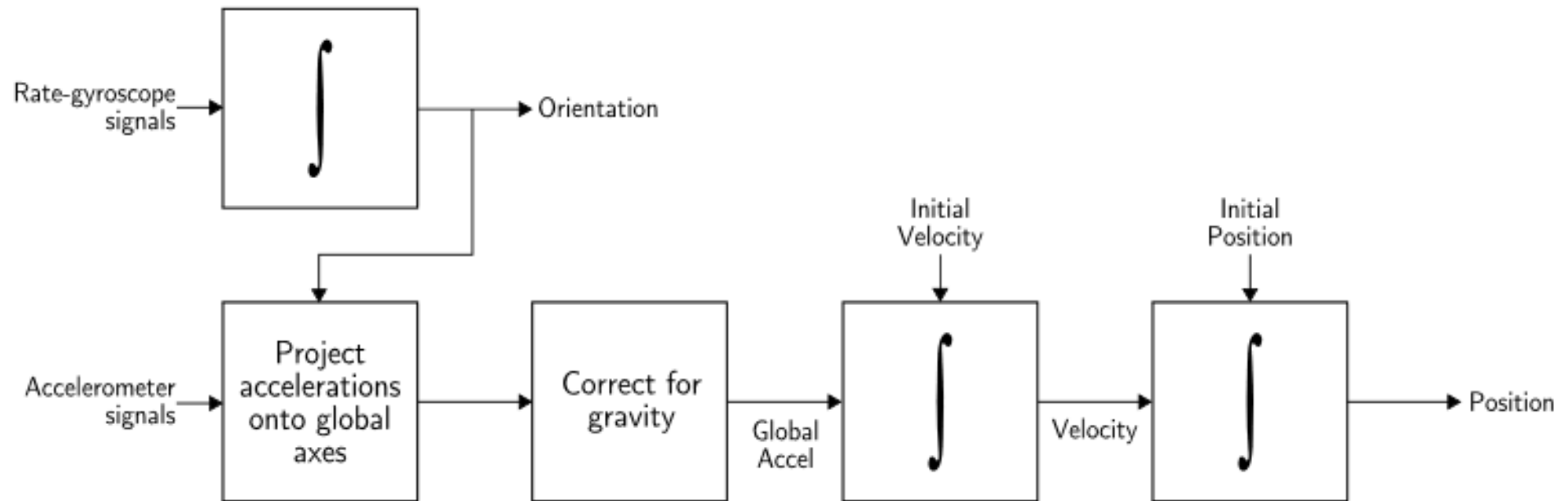


Dead-reckoning Options

Wheel ticks + steering wheel angle	Wheel ticks + Z gyroscope	Wheel ticks + Z gyroscope + 3 accelerometers	Wheel ticks + 6-axis IMU
• 5-10% accuracy • Two-dimensional dead reckoning with coarse heading • Ignores height	• 4-7% accuracy • Two-dimensional (no roll+pitch) • Gyroscope replaces steering wheel angle for finer heading resolution • Ignores height • Heavily dependent from vehicle frame alignment	• 2-5% • Accelerometers used for levelling and coarse roll/pitch estimation • Typically 2D, but can also provide coarse 3D from accels • Heavily dependent from vehicle frame alignment	• 1-3% • Accelerometers used for levelling and fine roll/pitch estimation • 3D navigation • Can estimate accelerometer biases from the gyroscope • Can correct vehicle frame alignment

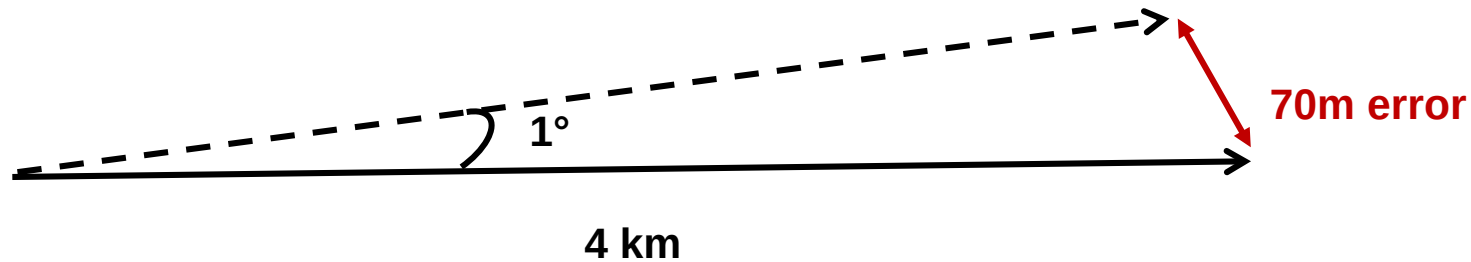
6-axis IMU increases accuracy up to 10x

Determining Speed, Position, Angle



Impact of Heading Estimation Error

1° heading error produces 1.75% error of distance traveled



Angle	Error
1°	1.75% of distance
2°	3.49% of distance
3°	5.24% of distance
5°	8.72% of distance
10°	17.4% of distance

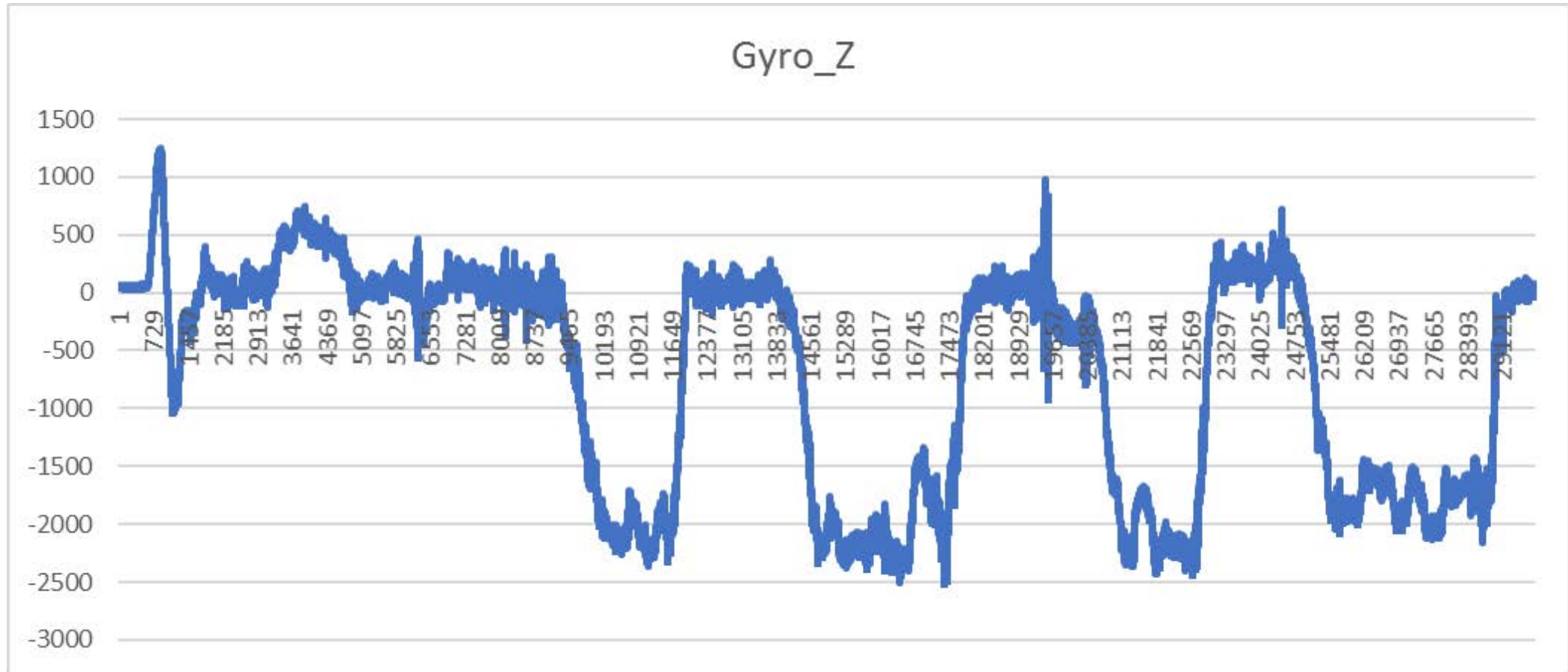
Impact of Acceleration Estimation Error

$$error = \int_0^T \left(\int_0^T \Delta a \, dt \right) dt = \frac{1}{2} \Delta a T^2$$

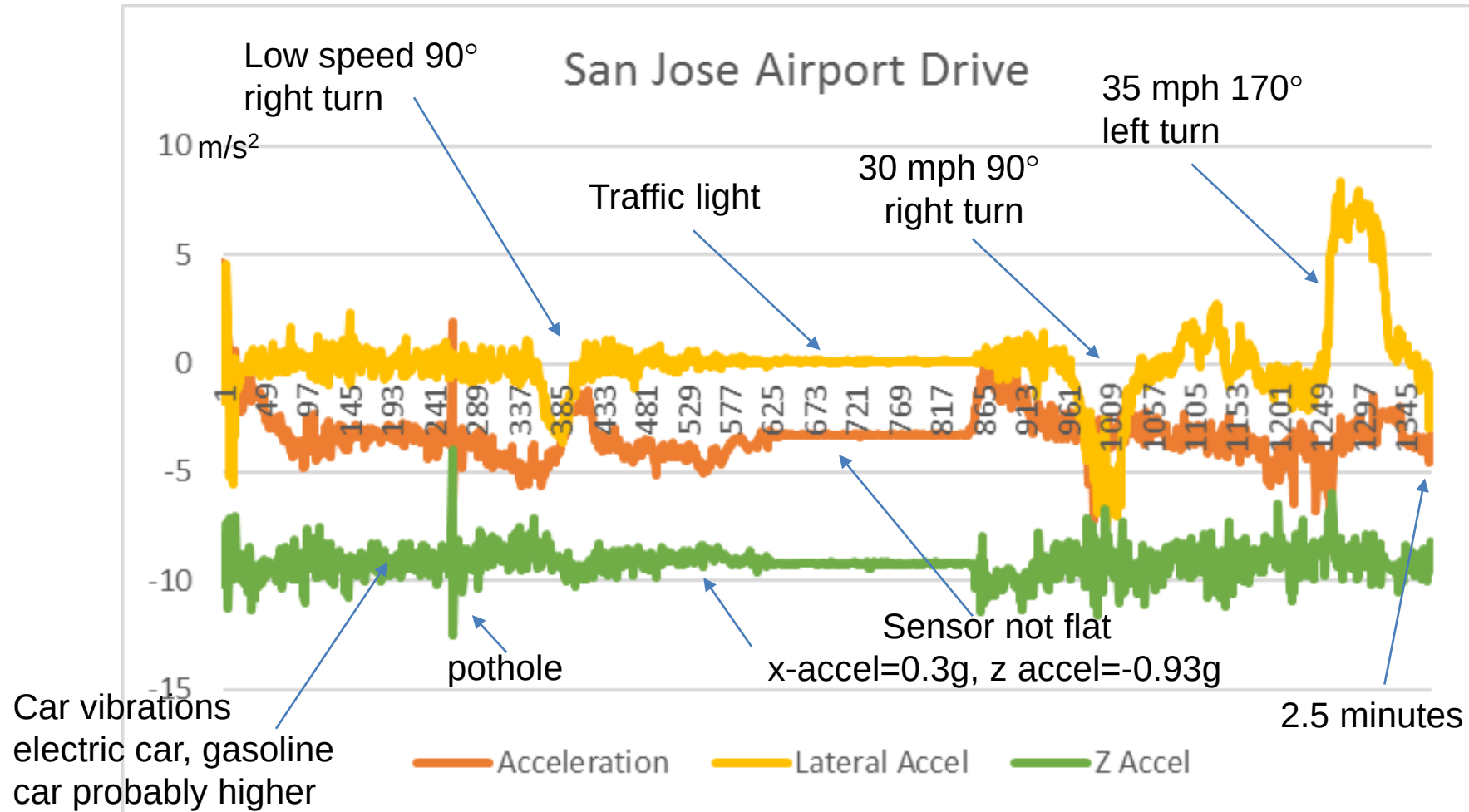
Every mg of constant offset error produces a distance error that grows with the square of time

Bias	Time	Speed	Distance	Error
1 mg	120s	120 km/h	4 km	71 m
5 mg	120s	120 km/h	4 km	353 m
10 mg	120s	120 km/h	4 km	706m
50 mg	120s	120 km/h	4 km	3.5 km
100 mg	120s	120 km/h	4 km	7 km

Example of Gyroscope Output (Sand Hill Drive)



Example of Accelerometer Output – San Jose Airport Drive

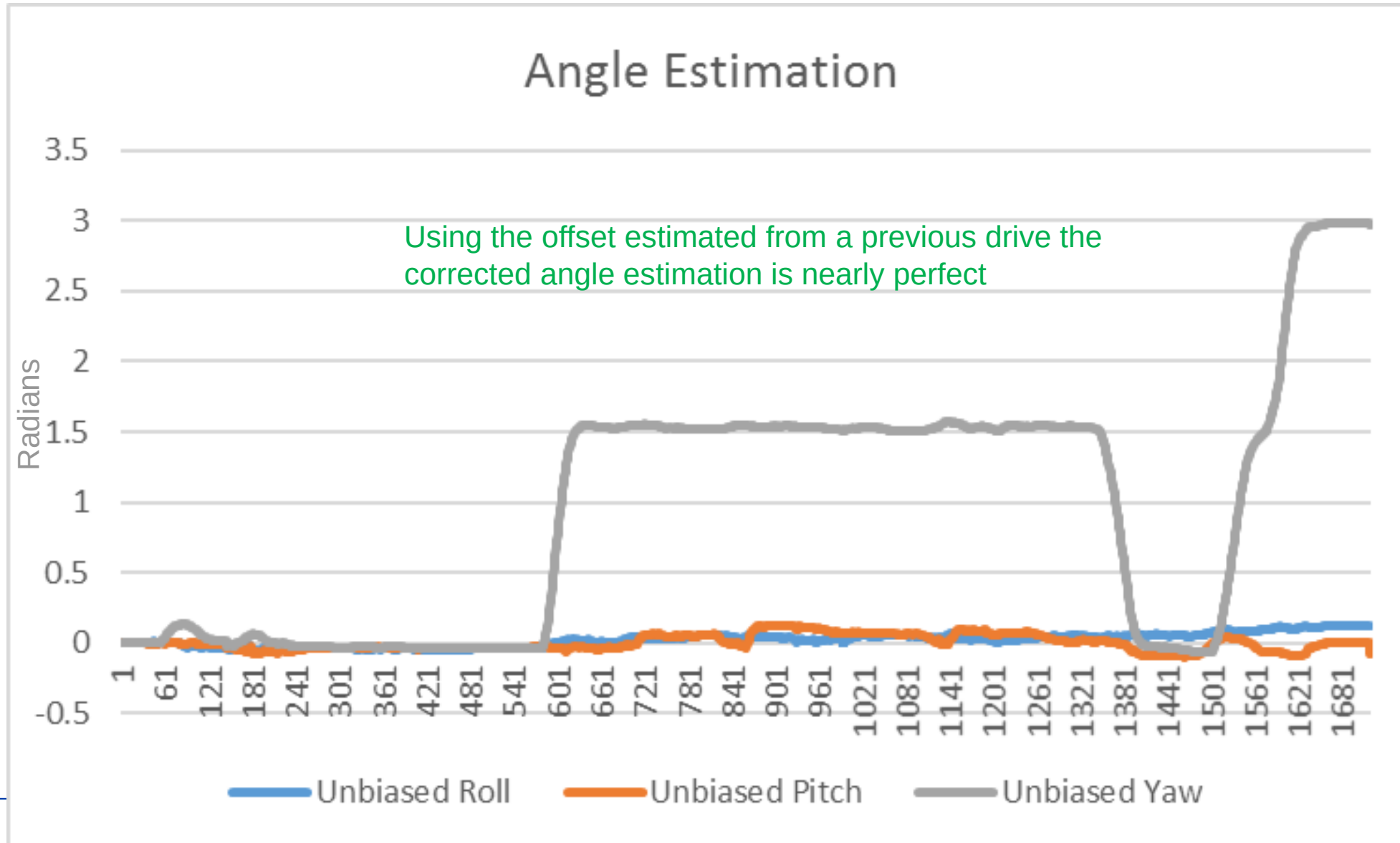


Lateral acceleration larger than forward acceleration

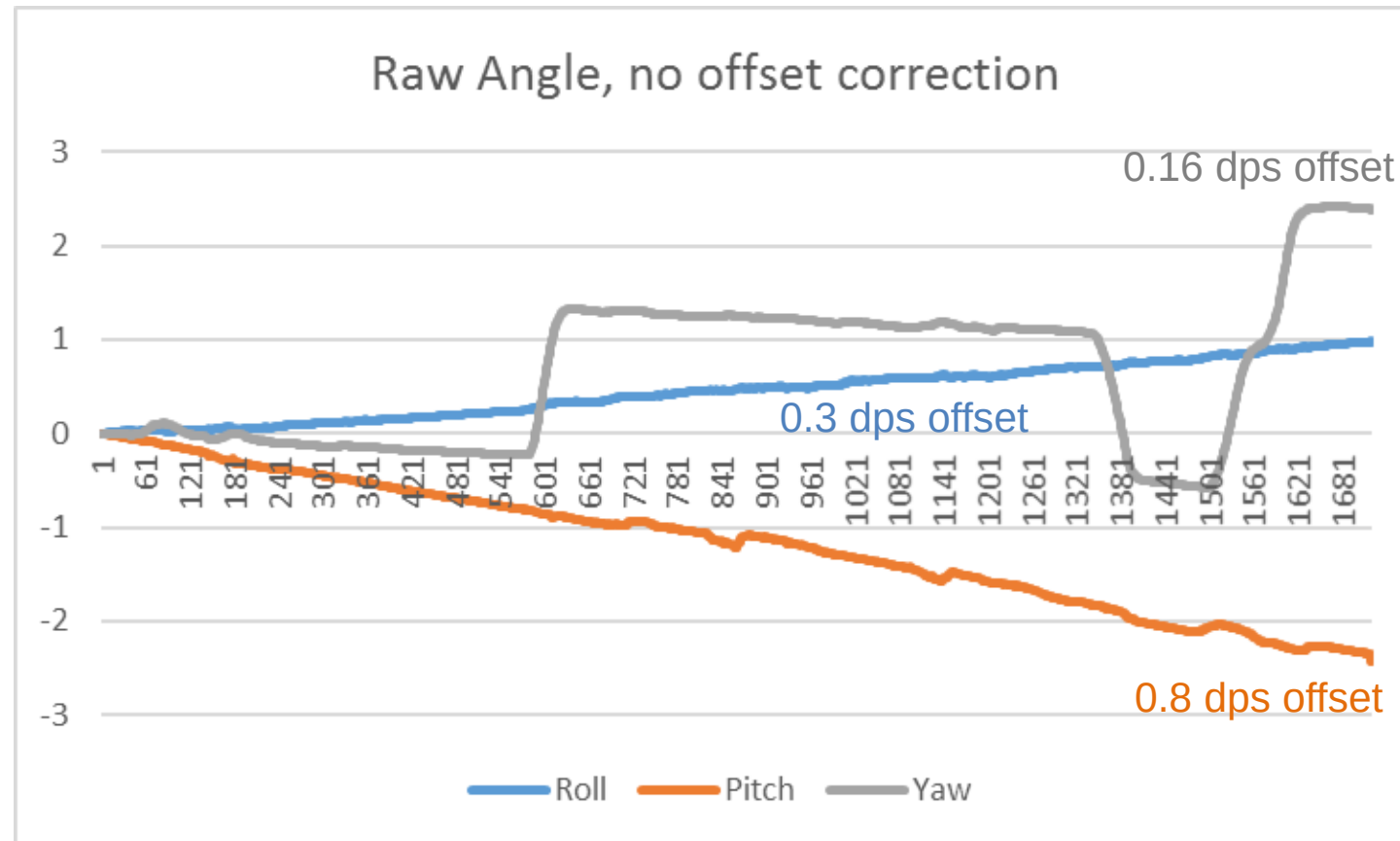
Effect of Gyroscope Errors - Uphill Drive in San Francisco



Offset Correction



Effect of a Small Constant Error



Even a tiny offset degrades reading very quickly => Need to constantly estimate offset

Gyroscope Noise

Angular random walk

- Standard deviation (σ) of the integral of the gyroscope output as function of time
 - ↳ Corresponds to an angle
 - ↳ Valid only in the -1/2 slope part of the Allan deviation plot
 - ↳ Appropriate for few seconds
 - ↳ Expressed in mdps/sqrt(Hz) or °/hr
- Example: 10 mdps/sqrt(Hz) -> 0.6°/sqrt(hr)
 - ↳ Angle drift in 10 seconds: 0.03°

Bias instability

- Bottom of the curve of the Allan deviation plot
 - ↳ Appropriate for minutes or hours
 - ↳ Expressed in °/hr
- Example: 12 °/hr
 - ↳ Angle drift in 2 minutes: 0.4°

Accelerometer Noise

Velocity random walk

- Standard deviation (σ) of the integral of the accelerometer output as function of time
 - ↳ Corresponds to a speed
 - ↳ Valid only in the -1/2 slope part of the Allan deviation plot
 - ↳ Appropriate for few seconds
 - ↳ Expressed in $\mu\text{g}/\sqrt{\text{Hz}}$
- Example: $500 \mu\text{g}/\sqrt{\text{Hz}}$
 - ↳ Velocity drift in 36 seconds: 30 mm/s

Accel bottom of the curve

- Bottom of the curve of the Allan deviation plot
 - ↳ Appropriate for minutes or hours
 - ↳ Expressed in mg or m/s^2
- Example: 0.1mg
 - ↳ Velocity drift in 2 minutes: 0.4 km/h

Effects of Bias vs. Temperature Error

- During dead-reckoning temperature might change by few degrees
- Accelerometer: 0.5-2 mg/°C
- Gyro: 0.05-0.4 dps/°C

ΔT	Time	Gyro Error	Angle Error	Accel Error	Distance Error
1°C	120s	0.05 dps/°C	6°	0.25mg/°C	18 m
	120s	0.2 dps/°C	24°	2mg/°C	71 m
2°C	120s	0.05 dps/°C	12°	0.25mg/°C	36 m
	120s	0.2 dps/°C	48°	2mg/°C	282 m
3°C	120s	0.05 dps/°C	18°	0.25mg/°C	53 m
	120s	0.2 dps/°C	72°	2mg/°C	424 m
4°C	120s	0.05 dps/°C	24°	0.25mg/°C	71 m
	120s	0.2 dps/°C	96°	2mg/°C	565 m

Effect of Gyroscope Sensitivity

Constant Sensitivity Error

$$\text{Error} = \frac{\Delta S}{S} \theta_{\text{Driven}}$$

θ_{Driven} is the sum of angles driven

Example: 1080° driven clockwise with a 1% sensitivity error yields 10.8° error

Example 2: 540° degrees driven clockwise and 540° counter-clockwise yields no error

Example 3: **straight line => no error due to sensitivity**

Different sensitivity turning clockwise and counterclockwise

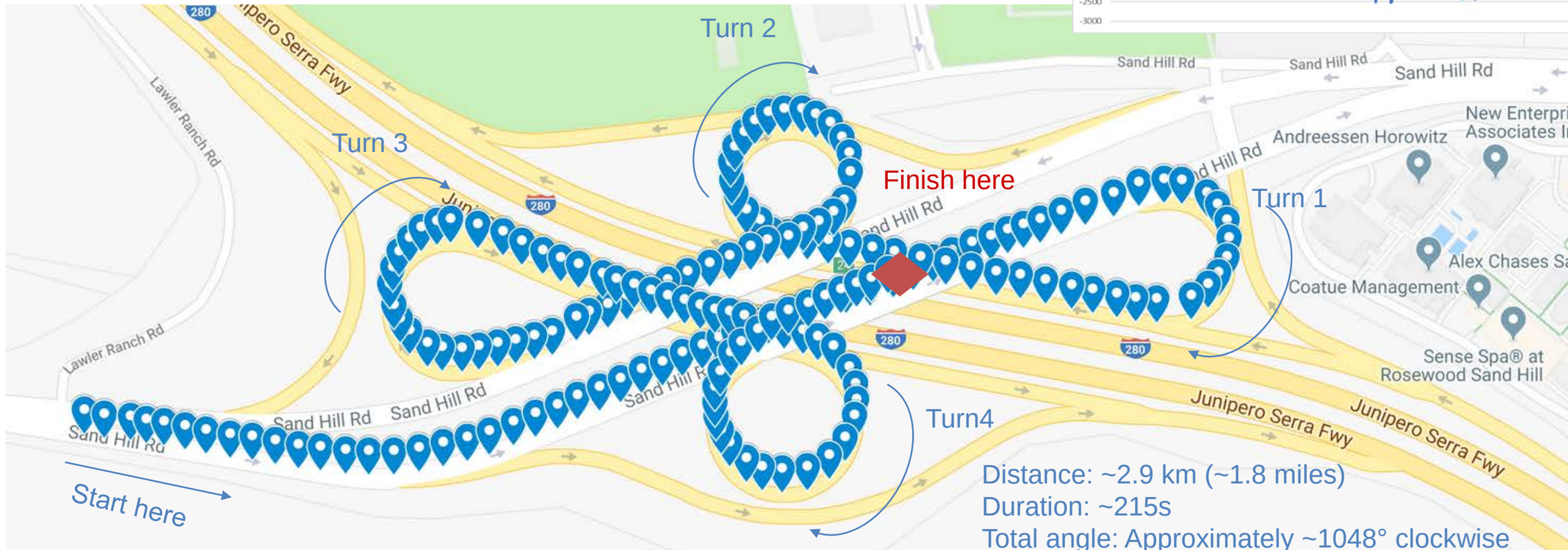
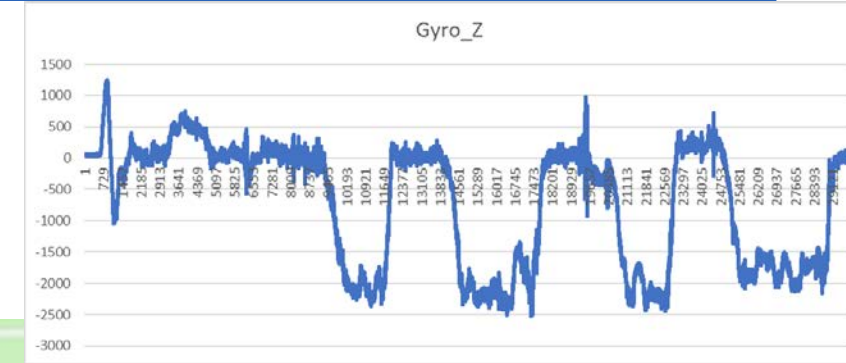
$$\text{Error} = \frac{\Delta S_+}{S} \theta_{\text{Counterclockwise}} + \frac{\Delta S_-}{S} \theta_{\text{Clockwise}}$$

Example 3: 540° degrees driven clockwise and 540° counter-clockwise yields an error that is $= (\frac{\Delta S_+}{S} - \frac{\Delta S_-}{S}) * 540$

$$\theta_{\text{measured}} = \int_0^T \omega(t) dt = \int_0^T (S + \Delta S) \text{Output}(t) dt = (1 + \frac{\Delta S}{S}) \int_0^T S \text{Output}(t) dt = (1 + \frac{\Delta S}{S}) \theta_{\text{Driven}}$$

Sand Hill – Slanted Clover Leaf

IAM-20680 Error: $< 3^\circ \Rightarrow$ Sensitivity Error $< 0.3\%$



Effects of Accelerometer Sensitivity

- Same math as gyroscope
- Typical number 3-5%

Sensitivity Error	Time	Distance	Distance Error
0.1%	120s	4 km	4 m
1%	120s	4 km	40 m
2%	120s	4 km	80 m
5%	120s	4 km	200 m
10%	120s	4 km	400 m

Pulling It Together

Dead reckoning typically lasts few seconds to few minutes

Parameter	Drive condition	Frequency of drive condition	Error magnitude
Gyro Noise	Straight line	Most common	Large
Accel Noise	Straight line	Most common	Small
Gyro Offset vs. T	T change	Infrequent	Largest
Accel Offset vs. T	T change	Infrequent	Medium
Gyro Sensitivity	Curve	Common	Large
Accel Sensitivity	Large acceleration	Infrequent	Medium

My ranking: Gyro noise, gyro offset vs. T, gyro sensitivity

Accelerometers, Gyroscopes, and IMUs

Near-term Motion Roadmap

More details
under NDA

6-axis

IAM-20680

In-cabin
3-axis gyro + 3-axis accel
Grade 3: -40°C to 85°C
Package: 3mmx3mm

3-axis gyro

IAM-20380

In-cabin
3-axis gyro
Compatible with IAM-20680
Grade 3: -40°C to 85°C
Package: 3mmx3mm

3-axis accel

IAM-20381

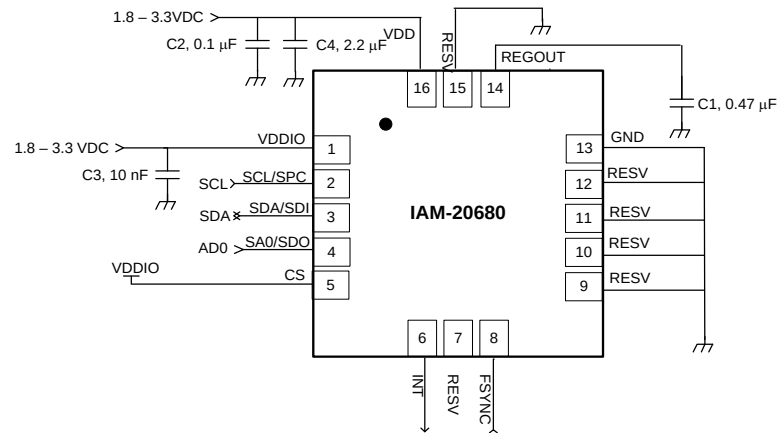
In-cabin
3-axis accel
Compatible with IAM-20680
Grade 3: -40°C to 85°C
Package: 3mmx3mm

- Smallest 6-axis footprint among released products
- Only package/pin/footprint/register compatible series in the market
- Best noise in the market

IAM-20680 6-Axis Automotive IMU

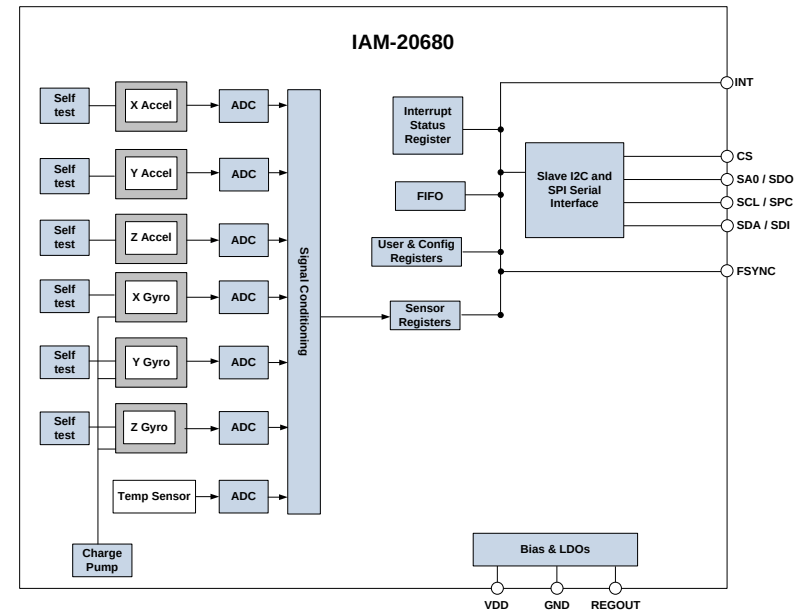
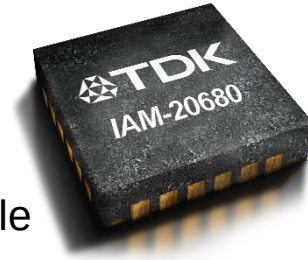
Features

- Highly accurate 3-axis accelerometer
- Highly accurate 3-axis gyroscope
- Low noise
- Temperature measurement sensor
- Low component count
- Programmable low pass filter
- Supports SPI and I2C
- Tested per AEC-Q100 (-40C to 85C)
- Small LGA package (3x3-0.75 mm)



Status

- In production
- Evaluation board available
- 100% burn-in
- Three-temperature testing
- PPAP available



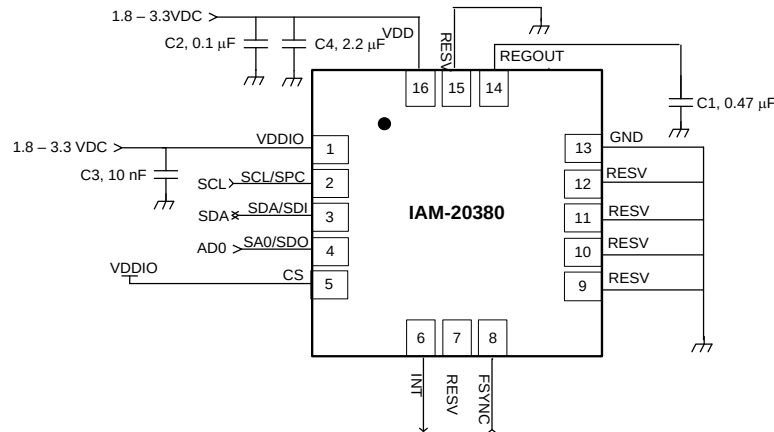
IAM-20680 Specifications

Accelerometer full scale	±2/4/8/16 g
Accelerometer sensitivity	2048-16384 LSB/g
Accelerometer resolution	16 bits
Gyroscope full scale	±250-2000 °/s
Gyroscope sensitivity	16.4-131 LSB/(°/s)
Gyroscope resolution	16 bits
Low pass filter	5-218 Hz
Power consumption	3 mA
Standby power consumption	6 µA
Output data rate	8 kHz
Temperature range	-40C to 85C

IAM-20380 3-Axis Automotive Gyroscope

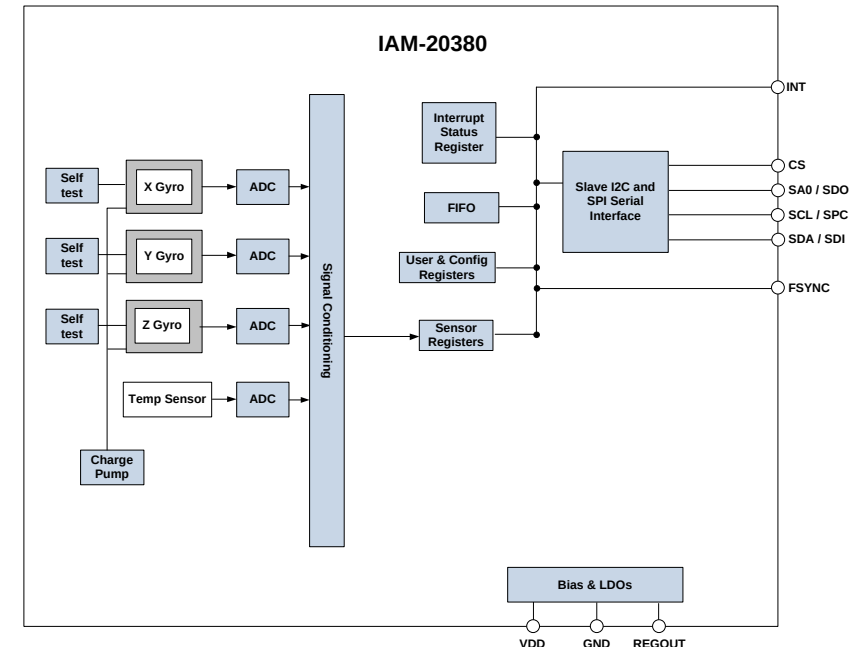
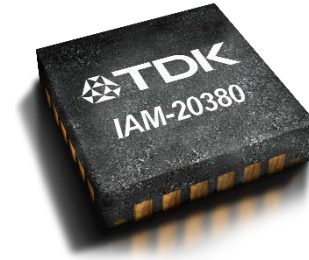
Features

- Highly accurate 3-axis gyroscope
- Low noise
- Temperature measurement sensor
- Low component count
- Programmable low pass filter
- Supports SPI and I2C
- Low power
- Tested per AEC-Q100 (-40C to 85C)
- Small LGA package (3x3-0.75 mm)



Status

- Datasheet available
- Samples available
- 100% burn-in
- Three-temperature testing



IAM-20380 Specifications

Gyroscope full scale	$\pm 250\text{-}2000\text{ }^{\circ}/\text{s}$
Gyroscope sensitivity	16.4-131 LSB/($^{\circ}/\text{s}$)
Gyroscope resolution	16 bits
Low pass filter	5-218 Hz
Power consumption	3 mA
Standby power consumption	6 μA
Output data rate	8 kHz
Temperature range	-40C to 85C

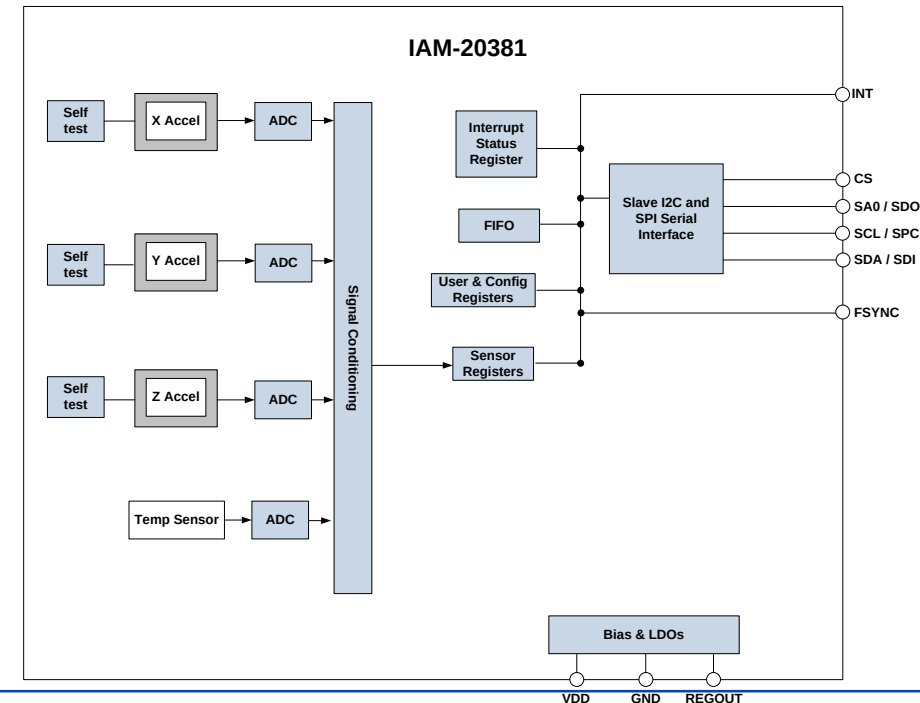
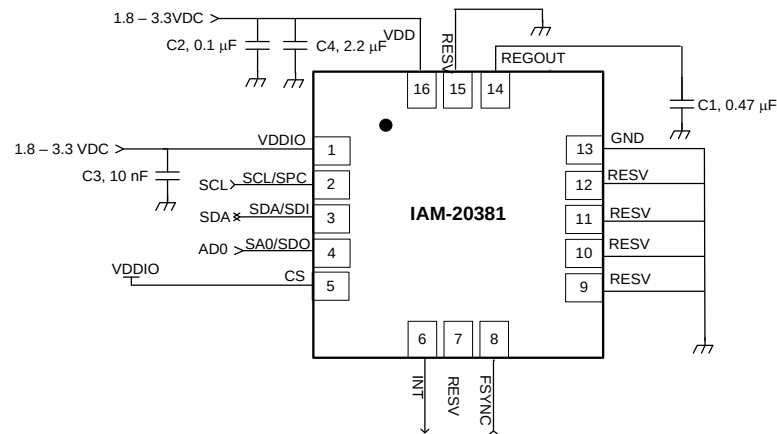
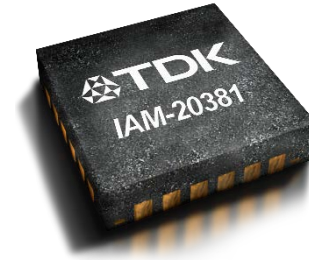
IAM-20381 3-Axis Automotive Accelerometer

Features

- Highly accurate 3-axis accelerometer
- Low noise
- Temperature measurement sensor
- Low component count
- Programmable low pass filter
- Supports SPI and I2C
- Low power
- Tested per AEC-Q100 (-40C to 85C)
- Small LGA package (3x3-0.75 mm)

Status

- Datasheet available
- Samples available
- 100% burn-in
- Three-temperature testing



IAM-20381 Specifications

Accelerometer full scale	±2/4/8/16 g
Accelerometer sensitivity	2048-16384 LSB/g
Accelerometer resolution	16 bits
Low pass filter	5-218 Hz
Power consumption	3 mA
Standby power consumption	6 µA
Output data rate	8 kHz
Temperature range	-40C to 85C



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Sand Hill Drive – Zero-sum Curves

Distance: ~3.9 km (~2.4 miles)

Duration: ~270s

Total angle: 0 (~570° clockwise and ~570° counterclockwise)

